Exercise 48

Find y' and y''.

$$y = \frac{1}{(1+\tan x)^2}$$

Solution

Take the derivative using the quotient rule and the chain rule.

$$\begin{aligned}
y' = \frac{dy}{dx} &= \frac{d}{dx}\left[\frac{1}{(1+\tan x)^2}\right] \\
&= \frac{\left[\frac{d}{dx}(1)\right](1+\tan x)^2 - \left[\frac{d}{dx}(1+\tan x)^2\right](1)}{(1+\tan x)^4} \\
&= \frac{(0)(1+\tan x)^2 - \left[2(1+\tan x)\cdot\frac{d}{dx}(1+\tan x)\right](1)}{(1+\tan x)^4} \\
&= \frac{-\left[2(1+\tan x)\cdot(\sec^2 x)\right](1)}{(1+\tan x)^4} \\
&= -\frac{2\sec^2 x}{(1+\tan x)^3}
\end{aligned}$$

Take another derivative.

$$\begin{aligned}
y'' = \frac{d}{dx}(y') &= \frac{d}{dx}\left[-\frac{2\sec^2 x}{(1+\tan x)^3}\right] \\
&= -\frac{d}{dx}\left[\frac{2\sec^2 x}{(1+\tan x)^3}\right] \\
&= -\frac{\left[\frac{d}{dx}(2\sec^2 x)\right](1+\tan x)^3 - \left[\frac{d}{dx}(1+\tan x)^3\right](2\sec^2 x)}{(1+\tan x)^6} \\
&= -\frac{\left[(4\sec x)\cdot\frac{d}{dx}(\sec x)\right](1+\tan x)^3 - \left[3(1+\tan x)^2\cdot\frac{d}{dx}(1+\tan x)\right](2\sec^2 x)}{(1+\tan x)^6} \\
&= -\frac{\left[(4\sec x)\cdot(\sec x\tan x)\right](1+\tan x)^3 - \left[3(1+\tan x)^2\cdot(\sec^2 x)\right](2\sec^2 x)}{(1+\tan x)^6} \\
&= -\frac{4\sec^2 x\tan x(1+\tan x)^3 - 6\sec^4 x(1+\tan x)^2}{(1+\tan x)^6} \\
&= -\frac{2\sec^2 x(1+\tan x)^2[2\tan x(1+\tan x) - 3\sec^2 x]}{(1+\tan x)^6} \\
&= -\frac{2\sec^2 x[2\tan x + 2\tan^2 x - 3\sec^2 x]}{(1+\tan x)^4}
\end{aligned}$$